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*Phase Diversity: a technique for Wave-Front
Sensing and for Diffraction-Limited Imaging*

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Abstract

The theoretical angular resolution of an optical imaging instrument such as a telescope is given by the ratio of the imaging wavelength λ over the aperture diameter D of the instrument.

For real-world instrument, optical aberrations often prevent this so-called diffraction-limit resolution λ/D from being achieved. These aberrations may arise both from the instrument itself and from the propagation medium of the light. The aberrations can be compensated either during the image acquisition by real-time techniques or a posteriori, i.e., by post-processing. Most of these techniques require the measurement of the aberrations, also called wave-front, by a wave-front sensor (WFS).

The focal-plane family of sensors was born from the very natural idea that an image of a given object contains information not only about the object, but also about the wave-front. A focal-plane sensor thus requires little or no optics other than the imaging sensor; it is also the only way to be sensitive to all aberrations down to the focal plane.

The first practical method for wave-front sensing from focal-plane data was proposed by Gerchberg and Saxton (1972). This so-called "phase-retrieval" method has two major limitations. Firstly, it only works with a point source. Secondly, there is generally a sign ambiguity in the recovered phase, i.e., the solution is not unique, as will be detailed below. Gonsalves (1982) showed that by using a second image with an additional known phase variation with respect to the first image (such as defocus), it is possible to estimate the unknown phase even when the object is extended and unknown. The presence of this second image additionally removes the above-mentioned sign ambiguity of the solution. This technique is referred to as "phase diversity"

This contribution attempts to provide a survey of the phase diversity technique, with an emphasis on its wave-front sensing capabilities.

Section 1 gives an introduction to the image formation for the considered instruments (i.e. those working with spatially incoherent light, such as telescopes), reviews the sources of image degradation, and states the inverse (estimation) problem to be solved in phase diversity. Section 2 reviews the domains of application of phase diversity. Then, Sections 3 and 4 review the wave-front estimation methods associated with this technique and their properties, while Section 5 examines the possible object estimation (i.e., image restoration) methods. Section 6 gives some background on the various minimization algorithms that have been used for phase diversity. Section 7 illustrates the use of phase diversity on experimental data for wave-front sensing. Finally, Sections 8 and 9 highlight two fields of phase diversity wave-front sensing that have witnessed noteworthy advances: Section 8 reviews the methods used to estimate the large-amplitude aberrations that one faces when imaging through turbulence, and proposes a novel approach for this difficult problem. And Section 9 reviews the developments of phase diversity for a recent application: the phasing (also called cophasing) of multi-aperture telescopes.

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