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APPROXIMATE ANALYTICAL EXPRESSION FOR AO-CORRECTED CORONAGRAPHIC IMAGING IN PREPARATION OF EXOPLANET SIGNAL EXTRACTION

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Abstract. The next step in the field of extrasolar planets imaging is on the verge of being reached with instruments such as SPHERE (Spectro Polarimetric High contrast Exoplanet REsearch), which will be capable of performing at the same time direct detection and spectral characterization thanks to integral field spectrograph (IFS) images.

In these multispectral images, the star is not completely cancelled by the AO-corrected coronagraphic system because of residual aberrations of the latter. In particular, the star image comprises quasi-static speckles that must be disentangled from the planet signal in order to get the sought information: is there a planet, where is it and what is its spectrum?

We are developing a specific image post-processing method using a Bayesian inverse problem approach. The essential required building block of such a method is a data model (often called "direct model") with a minimal number of unknown parameters. In the framework of the SPHERE project for the VLT, we propose an approximate analytical direct model of a long-exposure star image for an AO-corrected coronagraphic imaging system and we present some preliminary numerical simulations to validate this model.

Keywords: Exoplanets, Direct detection, Multispectral deconvolution, Inverse problems, Instrumentation, SPHERE, IFS

1 Introduction

Ground-based instruments have now demonstrated the capability to detect planetary mass companions (Chauvin et al. 2004; Lagrange et al. 2009; Marois et al. 2008; Kalas et al. 2008). First detections have been possible in favourable cases, at large separations and in young systems when low mass companion are still warm ($>=$ 1000 K) and therefore not too faint. There is a very strong astrophysical case to improve the high contrast detection capability very close to stars.

Thus, several instruments combining adaptive optics (AO) and coronagraphs are currently under construction. It is the case of SPHERE (Beuzit et al. 2010), GPI (Graham et al. 2007) and several others that will follow such as EPICS (Kasper et al. 2008). All these instruments will be capable of performing multispectral imaging and will allow characterizing the planets by measuring their spectra.

One of the main limitations for high contrast imaging is the presence of speckles in the focal plane (Racine et al. 1999). They find their origin in wave front imperfections and evolves on various time scale. In order to distinguish a planet from these speckles, it is important to modelize the speckle pattern in function of the aberrations in presence of a coronagraph and adaptive optics.

2 Envisaged post-processing method

In the case of multispectral imaging, some post-processing methods have already been proposed in order to overcome the problem of detection limitation caused by the non-static speckles. Thus, Sparks and Ford (2002)

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were the first to describe the so-called "spectral deconvolution" method in the framework of space-based observations for an instrument combining a coronagraph and an integral-field spectrograph. The goal of this method is to take advantage of the wavelength dependence of the PSF in order to remove the speckles while preserving both the flux and spectrum of the planet. The method is entirely based on the fit of a low-order polynomial. Latter, Thatte et al. (2007) presented an extension of the spectral deconvolution method to achieve very high contrast at small inner working radii for AO-corrected ground-based observations but without a coronagraph.

Nevertheless, the current spectral deconvolution method presents some limitations. It is an empirical method where no physical model is made explicit. Moreover, if some planets are present, they perturb the fit. That is why we propose an alternative approach, the Bayesian inverse problem (Idier 2008), which could estimate simultaneously the speckle field and the planet position by taking into account both the prior information we have on the speckles and the one we have on the possible planets.

3 Direct model

3.1 Model of coronagraphic imaging

In order to carry out the Bayesian inverse problem method, we need to derive a parametric direct model of coronagraphic imaging. We assume that, for an AO-corrected coronagraphic image, the direct model is the following sum of three terms, separating the coronagraphic stellar halo, the circumstellar source (for which the impact of coronagraph is neglected) and noise n_λ:

$$i_\lambda(x,y) = h^c_\lambda(x,y) + o_\lambda(x,y) \star h^{nc}_\lambda(x,y) + n_\lambda(x,y)\,, \tag{3.1}$$

where the data are: $i_\lambda(x,y)$, the image we have access to and $h^{nc}_\lambda(x,y)$, the non-coronagraphic PSF which can be calibrated separately. Solving the inverse problem is finding the unknowns: the object $o_\lambda(x,y)$ and the speckle field $h^c_\lambda(x,y)$ which is the coronagraphic "point spread function".

A model description of $h^c_\lambda(x,y)$ directly depends on the turbulence residuals and optical wave front errors. In case of coronagraphic imaging, it is important to distinguish pre-coronagraphic aberrations from post-coronagraphic aberrations. After previous works to model non coronagraphic PSFs (Perrin et al. 2003) and coronagraphic PSFs (Cavarroc et al. 2006; Soummer et al. 2007), Sauvage et al. (2010) proposed an analytical expression for coronagraphic image with a distinction between upstream and downstream aberrations. In the perspective of using such an expression as a basis for inversion, we derive and discuss the merits of an approximation of the Sauvage et al. (2010) expression.

3.2 Model of a non-perfectly corrected approximate coronagraphic PSF

We consider the optical system of Figure 1 of Sauvage et al. (2010)'s paper composed of a telescope, a perfect coronagraph and a detector plane. Some residual turbulent aberrations $\phi_r(\rho,t)$ are introduced in the telescope pupil plane. $\phi_r(\rho,t)$ is assumed to be temporally zero-mean, stationary, ergodic and with a power spectral density $S_{\phi_r}(\alpha)$. The static aberrations are separated into two contributions: the aberrations upstream of the coronagraph $\phi_u(\rho)$, in the telescope pupil plane $\mathcal{P}_u(\rho)$ and the aberrations downstream of the coronagraph $\phi_d(\rho)$ in the Lyot Stop pupil plane $\mathcal{P}_d(\rho)$. The perfect coronagraph is defined as an optical device that subtracts a centered Airy pattern to the electromagnetic field.

Assuming that all the phases are small and that the spatial mean of $\phi_u(\rho)$ and $\phi_d(\rho)$ are equal to zero on aperture, we derive a second-order Taylor expansion of expression 24 of Sauvage et al. (2010)'s paper:

$$h_c(\alpha) \quad = \quad \left|\widetilde{\mathcal{P}}_d(\alpha) \star \widetilde{\phi}_u(\alpha)\right|^2 + \left|\widetilde{\mathcal{P}}_d(\alpha)\right|^2 \star S_{\phi_r}(\alpha) - \left\langle \left|P\left[\phi_r\right](t)\right|^2 \right\rangle_t \cdot \left|\widetilde{\mathcal{P}}_d(\alpha)\right|^2 + o\left(\phi^2\right), \tag{3.2}$$

where $\widetilde{\mathcal{P}}_d(\alpha)$ and $\widetilde{\phi}_u(\alpha)$ are the Fourier transforms of the downstream pupil and upstream aberrations respectively and $P\left[\phi_r\right](t)$ denotes the piston of phase $\phi_r(\rho,t)$. $\left\{ \left\langle \left|P\left[\phi_r(\alpha,t)\right]\right|^2 \right\rangle_t \cdot \left|\widetilde{\mathcal{P}}_d(\alpha)\right|^2 \right\}$ is a corrective term that compensates for the fact that $\phi_r(\rho,t)$ is stationary and thus non-piston-free on the aperture at every instant. Note that $\left|\widetilde{\mathcal{P}}_d(\alpha)\right|^2$ is the Airy pattern formed by the pupil $\mathcal{P}_d(\alpha)$.

4 First results of validation

Qualitative validation This expression is far more intuitive than Equation (24) of Sauvage et al. (2010) and brings physical insight to it:

- firstly, the downstream aberrations do not appear in this expression. While the exact expression depends on three parameters ($\phi_u(\rho)$, $\phi_d(\rho)$ and $D_{\phi_r}(\rho)$), the approximate expression depends on only two parameters which are the Fourier transform of the static upstream phase aberrations $\widetilde{\phi}_u(\alpha)$ and the PSD of the residual turbulent aberrations $S_{\phi_r}(\alpha)$. Cavarroc et al. (2006) had already showed that the main limitation comes from the static aberrations, now we have analytical confirmation that the static upstream aberrations are predominant with respect to the static downstream aberrations;
- secondly, we can separate our expression into two terms: one static term and one turbulent term which is coherent with the Soummer et al. (2007)'s paper and makes a connection between this paper and the physical parameters of interest $\widetilde{\phi}_u(\alpha)$ and $S_{\phi_r}(\alpha)$;
- thirdly, we can show that if we rewrite Equation (3.2) as a function of wavelength we can validate theoretically the spectral deconvolution method.

Quantitative validation We want to compare the approximate expression of Equation (3.2) to the exact expression of Equation (24) in Sauvage et al. (2010) to determine the error due to the Taylor expansion. The simulated instrumental conditions are typical of a SPHERE-like instrument (Fusco et al. 2006) and the same as these of Sauvage et al. (2010). Three simulation programs were used:

- the first simulates an empirical long exposure PSF by adding one hundred short-exposure coronagraphic PSF's;
- the second simulates the analytical long exposure coronagraphic image model;
- the third simulates the approximate analytical long exposure coronagraphic image model.

In order to test the approximate expression for each kind of aberration and to quantify the errors due to the Taylor expansion, we compared it with the exact expression. The results of this comparison are shown in Figure 1. We plotted the circularly averaged intensity profiles of the exact model, the approximate model and the circularly averaged root mean square residual of the difference between them in different configurations to quantify the influence of each kind of aberration:

- in presence of all kinds of aberrations, the corresponding error is about twenty-nine percent (Figure 1, left);
- because the approximate model do not take the downstream aberrations into account, it is interesting to compute the same results without downstream aberrations for the exact model. The corresponding error is about seventeen percent. That means that the contribution of the downstream aberrations is about less than a third of the total error (Figure 1, right);
- if we do the same, without residual turbulent aberrations, the corresponding error is about seven percent. Here again, we can deduce the contribution of the residual turbulent aberrations and upstream aberrations which are approximately equal for these simulation conditions representative of SPHERE (Figure 1, right).

It is interesting to see that the three kinds of aberrations have approximately the same non-negligible error. We must be careful with this result which is right for the SPHERE instrument where the downstream aberrations are three times more important than the upstream aberrations. Thus, it does not call the fact that the upstream aberrations are predominant besides the downstream aberration into question.

These results raise some interesting questions as for example: What happens if we consider a non perfect coronagraph? How do these errors propagate and how do they affect the detection and characterization performances if we use our approximate expression in an inversion process?

5 Conclusion and prospects

We have obtained an approximate expression for AO coronagraphic image that is much simpler than the exact expression of Sauvage et al. (2010). Because it has one less parameter, it would be easier to implement in an inversion process. Qualitatively, the result is consistent with the previous works on coronagraphic and non-coronagraphic imaging. We also have discussed quantitatively the error level with respect to the exact expression due to the approximation in SPHERE-like conditions.

We are currently finalizing the numerical validation of the approximate expression by comparing the results obtained with a perfect 4-quadrant coronagraph and by propagating the aberrations errors into the inversion process. This will allow us to decide if we will use this approximate expression or the exact expression to perform the inversion. Then, we shall implement a first inversion on simulated images before adapting our inversion to real images from the SPHERE instrument.

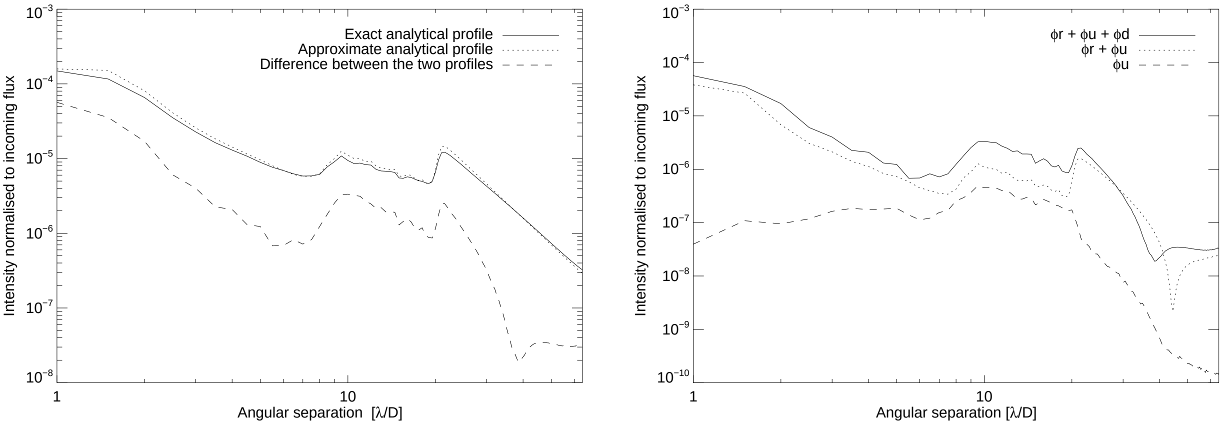

Figure 1. Circularly averaged profiles. [Left] Exact analytical model versus Taylor expansion and error between the two models. [Right] Evolution of the error between the two models in function of the kind of aberrations introduced.

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