

A. Gradients expression

The numerical minimization of criterion J (Eq. (3)) requires the analytic expression of gradients $\frac{\partial J}{\partial \phi_u}$, $\frac{\partial J}{\partial \phi_d}$, $\frac{\partial J}{\partial \alpha}$ and $\frac{\partial J}{\partial \beta}$ to estimate the aberrations upstream ϕ_u and downstream ϕ_d of the coronagraph, as well as the incoming flux α and the residual background β . Let us rewrite here the expression of criterion J :

$$\begin{aligned} J(\alpha, \beta, \phi_u, \phi_d) &= \frac{1}{2} \left\| \frac{i_c^{\text{foc}} - (\alpha_{\text{foc}} h_{\text{det}} \star h_c^{\text{foc}} + \beta_{\text{foc}})}{\sigma_n^{\text{foc}}} \right\|^2 + \frac{1}{2} \left\| \frac{i_c^{\text{div}} - (\alpha_{\text{div}} h_{\text{det}} \star h_c^{\text{div}} + \beta_{\text{div}})}{\sigma_n^{\text{div}}} \right\|^2 \\ &+ \mathcal{R}(\phi_u) + \mathcal{R}(\phi_d) \\ &= J^{\text{foc}} + J^{\text{div}} + \mathcal{R}(\phi_u) + \mathcal{R}(\phi_d) \end{aligned} \quad (20)$$

The expressions of $\frac{\partial J}{\partial \alpha}$ and $\frac{\partial J}{\partial \beta}$ can be found in [15]. The calculation of gradients $\frac{\partial J}{\partial \phi_u}$ and $\frac{\partial J}{\partial \phi_d}$ is performed following what have been done in [15]: we derive J^{foc} , and then deduce the gradients' expressions of J^{div} using a trivial substitution. The notations used here are the ones introduced in Section 2:

$$\begin{aligned} \frac{\partial J^{\text{foc}}}{\partial \phi_d} &= 2\Im \left\{ \psi_0^* - \varepsilon \psi_d \mathcal{F} [\mathcal{M} \mathcal{F}^{-1}(\psi_u)]^* \times \mathcal{F} \left[\frac{\partial J^{\text{foc}}}{\partial h_c^{\text{foc}}} (\Psi_0 - \varepsilon \Psi_c) \right] \right\} \\ \frac{\partial J^{\text{foc}}}{\partial \phi_u} &= 2\Im \left\{ \psi_0^* \mathcal{F} \left[\frac{\partial J^{\text{foc}}}{\partial h_c^{\text{foc}}} (\Psi_0 - \varepsilon \Psi_c) \right] \right. \\ &\quad \left. - \varepsilon \psi_u^* \mathcal{F} \left[\mathcal{M}^* \mathcal{F}^{-1} \left(\Psi_d^* \mathcal{F} \left\{ \frac{\partial J^{\text{foc}}}{\partial h_c^{\text{foc}}} [\Psi_0 - \varepsilon \Psi_c] \right\} \right) \right] \right\} \end{aligned} \quad (21)$$

with:

$$\frac{\partial J^{\text{foc}}}{\partial h_c^{\text{foc}}} = \frac{1}{\sigma_n^{\text{foc}2}} [\alpha h_{\text{det}} (\alpha h_{\text{det}} \star h_c^{\text{foc}} - i_c^{\text{foc}})] \quad (22)$$

and:

$$\begin{aligned} \psi_u &= P_u e^{j\phi_u} \\ \psi_d &= P_d e^{j\phi_d} \quad \Psi_d = \mathcal{F}^{-1}(\psi_d) \\ \psi_0 &= P_u e^{j(\phi_u + \phi_d)} \quad \Psi_0 = \mathcal{F}^{-1}(\psi_0) \\ \Psi_c &= \mathcal{F}^{-1} \{ \psi_d \mathcal{F} [\mathcal{M} \mathcal{F}^{-1}(\psi_u)] \} \end{aligned} \quad (23)$$

The regularization metric expression $\mathcal{R}(\phi_k)$ (k is for u (upstream) or d (downstream)) is given by Eq. (6). Its gradient $\frac{\partial \mathcal{R}}{\partial \phi_k}$ can be written as:

$$\frac{\partial \mathcal{R}}{\partial \phi_k} = \mu_k \|\Delta \phi_k(r)\|. \quad (24)$$

where Δ represent the Laplacian operator.