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DECONVOLUTION OF ADAPTIVE OPTICS CORRECTED IMAGES

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Abstract

Deconvolution is a necessary tool for the exploitation of adaptive optics corrected images, because the correction is partial. The Maximum *A Posteriori* (MAP) framework is used to derive a deconvolution method that combines the data with our knowledge of the noise statistics as well as our prior information about the object and the variability of the Point Spread Function. The deconvolution of experimental data illustrates the capabilities of this method.

Keywords: adaptive optics, atmospheric turbulence, deconvolution, image restoration, inverse problems, astronomy.

1 Introduction

The performance of high resolution imaging with large astronomical telescopes is severely limited by the atmospheric turbulence. Adaptive optics (AO) offers a real time compensation of the turbulence. The correction is however only partial and the long exposure images must be deconvolved to restore the fine details of the object.

A great care must be taken in the deconvolution process in order to obtain a reliable restoration with a good photometric precision because of the inevitable noise in the images. A key point is to recognize the fact that noise makes it necessary to add some prior knowledge on the observed object into the deconvolution method; failure to do so usually results in unacceptable amplification of the noise (Demoment 1989). The fact that the residual point spread function (PSF) is usually not perfectly known (Conan et al. 2000, Fusco et al. 1999) adds to the difficulty.

In this paper, we present a deconvolution method that falls within the Maximum *A Posteriori* (MAP) framework and that addresses these two points. It uses a prior well-adapted to astronomical objects that are a mix of sharp structures and smooth areas, such

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as planets and asteroids. It also estimates the PSF given some prior information on the average PSF and its variability. The implementation of this method is called MISTRAL (for Myopic Iterative STep-preserving Restoration ALgorithm) and takes into account the presence of a mixture of photon and electronic noises.

2 Partially Corrected AO Images

Within the isoplanatic angle, the intensity $\mathbf{i}(r)$ at the focal plane of the system consisting of the atmosphere, of the telescope and of the AO bench is given by:

$$\mathbf{i}(r) = \mathbf{h}(r) \star \mathbf{o}(r) + \mathbf{n}(r), \tag{2.1}$$

where r is the spatial coordinate, $\mathbf{o}(r)$ is the observed object, $\mathbf{h}(r)$ is the system PSF and $\mathbf{n}(r)$ is an additive zero mean noise.

We consider here the case of AO corrected long exposure images. The deconvolution procedure needs a measurement of the PSF. The usual procedure consists in recording the corrected image of a nearby unresolved star shortly after observing the object of interest. Since the correction quality depends on the observing conditions (turbulence strength, magnitude of the source used for wavefront sensing), the unresolved star image is not a perfect measurement of the PSF associated with the image to be deconvolved (Conan et al. 1998). Actually the main source of PSF variability is the seeing fluctuation.

3 Deconvolution Approach

Most deconvolution techniques boil down to the minimization (or maximization) of a criterion. The first issue is the definition of a suitable criterion for the given inverse problem. The criteria presented here will be derived from a well-known probabilistic approach detailed below. The second issue is then to find the position of the criterion's global minimum which is defined as the solution. In some cases it is given by an analytical expression, but most of the time one must resort to an iterative numerical method.

In the following sections, we first consider the case of an assumedly known PSF, so-called "classical" deconvolution; the method is then extended to the joint estimation of the object and the PSF, called here "myopic" deconvolution.

3.1 Deconvolution with known PSF

Following the probabilistic (Bayesian) approach called maximum *a posteriori* (MAP), the deconvolution problem can be stated as follows: we look for the most likely object $\mathbf{o}$ given the observed image $\mathbf{i}$ and our prior information on $\mathbf{o}$, which is summarized by a probability density $p(\mathbf{o})$. This reads:

$$\hat{\mathbf{o}}_{\text{map}} = \arg\max_{\mathbf{o}} p(\mathbf{o}|\mathbf{i}) = \arg\max_{\mathbf{o}} p(\mathbf{i}|\mathbf{o}) \times p(\mathbf{o}) = \arg\min_{\mathbf{o}} \left[J_n(\mathbf{o}) + J_o(\mathbf{o})\right]. \tag{3.1}$$

The criterion to be minimized, $J = J_n + J_o$, is composed of a first term ($J_n = -\ln p(\mathbf{i}|\mathbf{o})$) accounting for the noise statistics in the image, plus a second term ($J_o =$

$-\ln p(\mathbf{o})$) which incorporates the prior knowledge one has on the object. This second term should of course be dependent on the type of object being observed.

If no prior knowledge is used, which corresponds to setting $p(\mathbf{o})$ =constant in the above equation, one then maximizes $p(\mathbf{i}|\mathbf{o})$ (likelihood of the data) so that the solution is a maximum likelihood solution. In this case the criterion is only constituted of the term J_n. The Richardson-Lucy algorithm is an example of an iterative algorithm which converges towards the minimum of J_n when the noise follows Poisson statistics. It is however well known that the restoration of the object using the sole data is an unstable process (Demoment 1989).

In this paper, we consider that the noise is non stationary white Gaussian, which is a good approximation of a mix of photon and background (detector or sky-background) noise; furthermore, the deconvolution is regularized by an object prior particularly adapted for planetary-like objects. This prior avoids the usual ringing artifacts given by standard deconvolution techniques on sharp edge objects. The corresponding expressions for J_n and J_o can be found in (Conan et al. 2000, Mugnier et al. 2001).

We use a conjugate gradient method to minimize the global criterion $J_n + J_o$. This method is well adapted since the so-defined criterion is convex. An additional positivity constraint is enforced with a reparameterization method ($\mathbf{o} = \mathbf{a^2}$) where $\mathbf{a}$ are the new parameters used in the minimization.

3.2 Myopic deconvolution

As mentioned in Sect. 2 the true residual PSF is seldom available. MISTRAL has the ability to estimate both the object and PSF from the image and some imprecise PSF measurement. Equation 3.1 can indeed be generalized, in the same probabilistic framework, to the case of a joint estimation of $[\mathbf{o}, \mathbf{h}]$. One obtains:

$$[\hat{\mathbf{o}}, \hat{\mathbf{h}}] = \arg\max_{\mathbf{o},\mathbf{h}} p(\mathbf{o},\mathbf{h}|\mathbf{i}) = \arg\max_{\mathbf{o},\mathbf{h}} p(\mathbf{i}|\mathbf{o},\mathbf{h}) \times p(\mathbf{o}) \times p(\mathbf{h})$$
$$= \arg\min_{\mathbf{o},\mathbf{h}} \left[J_n(\mathbf{o},\mathbf{h}) + J_o(\mathbf{o}) + J_h(\mathbf{h})\right]. \qquad (3.2)$$

The myopic criterion contains the two terms of Eq. 3.1, the first one now being a function of $\mathbf{o}$ and of $\mathbf{h}$, plus an additional term $J_h = -\ln p(\mathbf{h})$ which accounts for the knowledge, although partial, available on the PSF. Assuming stationary Gaussian statistics for the PSF, J_h is actually the energy of a set of springs (one per spatial frequency), bringing the OTF towards the mean OTF with a stiffness given by the power spectral density (PSD) of the PSF, which characterizes the variability of the OTF at the particular frequency (Conan et al. 1998).

3.3 Application to experimental data

MISTRAL has been applied to infrared images of Uranus acquired on May 2nd 1999 with the ADONIS AO system. The deconvolved images in J and H band exhibit structures on the planet (bright polar haze). When looking at low intensity levels (see Fig. 1) one can also see the structure of the Epsilon ring and of the innermost ones, as well as very faint

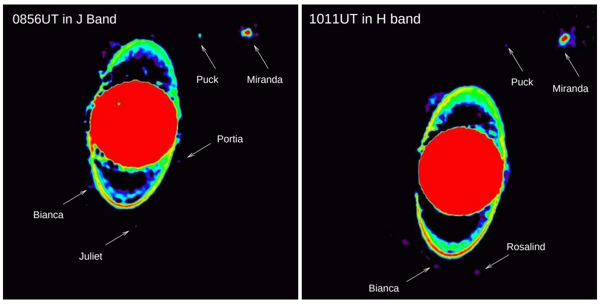

Fig. 1. Logarithmic display of J and H band images of Uranus deconvolved by MISTRAL. The planet surface is saturated to enhance low light levels.

satellites discovered by Voyager 2 in 1986 and never reobserved since. The high dynamic range and the photometric accuracy of MISTRAL therefore allows a precise monitoring of such planetary objects with Earth-bound telescopes.

3.4 Conclusion

The Maximum *A Posteriori* framework has been used to derive a deconvolution method that combines the data with our knowledge of the noise statistics as well as our prior information about the object and the variability of the Point Spread Function. A deconvolution result of experimental data has been presented to illustrate the capabilities of this method.

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