On-axis conoscopic holography without a conjugate image

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We present a method for removing the conjugate image in an incoherent-light holographic technique, namely, on-axis conoscopic holography. The point-spread function that we obtain is that of a complex Gabor zone pattern, which thus should allow good-quality reconstructions of objects. Experimental results are also presented, which confirm the validity of this method.

Conoscopic holography is a recent incoherent-light holographic technique first presented in 1985 (Ref. 1) that is based on the propagation of light in a birefringent medium. The basic setup is shown in Fig. 1: a uniaxial crystal (C) is sandwiched between two circular polarizers (P1, P2). A lens images the object—here a point source—into the system. When the monochromatic light from the image (S) of the point source passes through the crystal and the two polarizers, a Gabor zone pattern (GZP) is observed at the output; it is the result of the velocity disparity between the ordinary and the extraordinary waves in the crystal. This interference pattern can be recorded on a photographic plate and reconstructed optically[1] both coherently and incoherently. It can also, as in Ref. 2 and this Letter, be recorded on a CCD camera and digitized in a microcomputer for a numerical reconstruction of the image and the shape of the object. Such patterns thus are referred to as holograms.

In the on-axis (or in-line) configuration, the crystal axis is parallel to the geometrical axis, Oz, of the system, and the point-spread function (PSF), say H_c^+, is a real GZP plus a bias. For a complete object, the hologram is the incoherent superposition of the GZP of each point.

The two major problems in the reconstruction of the original object are the bias and the conjugate image. The bias is a usual drawback of incoherent holography; this problem has been solved[2,3] by inserting an electrically driven half-wave plate [a liquid-crystal light valve (LCLV)] after the first circular polarizer so as to change its handedess (e.g., from right to left handed), thus providing us with a second PSF, which we denote H_c^-. By calculating numerically the difference between the holograms obtained with each impulse response, we obtain a real GZP without bias, $H_c = H_c^+ - H_c^-$. These PSF's are given by

$$H_c^+(x, y) = \frac{1}{2}\{1 + \cos[\pi f_r(x^2 + y^2)]\}, \quad (1)$$

$$H_c^-(x, y) = \frac{1}{2}\{1 - \cos[\pi f_r(x^2 + y^2)]\}, \quad (2)$$

$$H_c = H_c^+ - H_c^- = \cos[\pi f_r(x^2 + y^2)], \quad (3)$$

where x and y are the coordinates in the recording plane and f_r is the Fresnel parameter, a scale factor that depends on the distance between the point and the recording plane. The exact expression of f_r can be found in Ref. 4, but it is not needed here. It is well known that the cosine in H_c can be decomposed into two exponentials, corresponding to the virtual image and to the conjugate real image of the point,[5] which, from an algorithmic point of view, is due to the fact that the real GZP H_c has zeros in the Fourier domain, whereas a phase GZP has none.

We address here the remaining problem, namely, the elimination of the conjugate image. The configurations used until now to remove it are (i) the quasi-complex configuration,[2,3] in which one of the polarizers is changed from circular to linear, whose PSF can be algorithmically modified to approximate a complex GZP, and (ii) the off-axis configuration, obtained by tilting the crystal axis[6] and/or by placing a stop in the front focal plane of the lens.[7–9] This configuration is only similar to and not identical with the coherent off-axis configuration. This leads to inherent drawbacks of the method that make it suitable for some applications such as a range finder[10] but not as a three-dimensional camera.

The method that we present here is inspired by these two configurations and yields directly an impulse response that is an on-axis phase GZP,

$$H_e(x, y) = \exp[i\pi f_r(x^2 + y^2)]. \quad (4)$$

The basic idea to remove this conjugate image is to modify again the impulse response by replacing the input circular polarizer with a linear one, as in the quasi-complex configuration, and by placing a mask in the front focal plane of the lens that images the object into the system (see Fig. 2). It can be shown by geometrical optics that if the mask transmittance depends on the sole polar angle θ of the recording plane, it is applied to the impulse response in a three-dimensional invariant way.

The first operation is done by setting the LCLV delay to a quarter-wave; the PSF is then

$$H(x, y) = \frac{1}{2}\{1 + \sin[2(\theta - \phi_0)]\sin[\pi f_r(x^2 + y^2)]\}, \quad (5)$$

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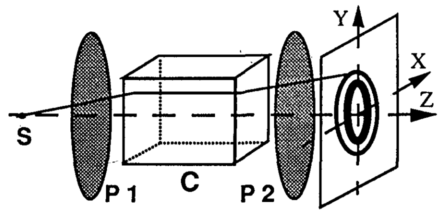

Fig. 1. Basic setup. A uniaxial crystal (C) is sandwiched between two circular polarizers (P1, P2). A point source (S) illuminates the system, and a Gabor zone pattern is observed at the output.

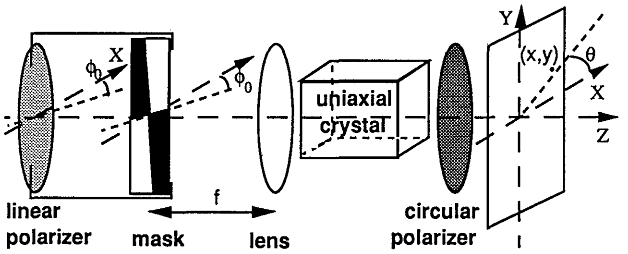

Fig. 2. Principle of the modified setup. The input polarizer is now linear and makes an angle ϕ_0 with axis X. A mask is added to this polarizer to modify the impulse response and is integral with it.

where ϕ_0 is the angle of the linear polarizer with the reference axis Ox of the recording plane. We remove the bias by changing the angle of the linear input polarizer from ϕ_0 to $\phi_0 + \pi/2$, which can be achieved as above by adding a half-wave retardation after the first polarizer (i.e., by setting the LCLV delay to a three-quarter wave) and by calculating the difference of the two holograms numerically. With a mask $m(\theta)$ installed, we record, after subtraction of the bias,

$$H(x, y) = m(\theta)\sin[2(\theta - \phi_0)]\sin[\pi f_r(x^2 + y^2)]. \quad (6)$$

Various masks are possible for removing the $\sin(2\theta)$ modulation and obtaining the imaginary part of the desired PSF, among which is the following binary amplitude mask (Fig. 3):

$$m(\theta) = 1 \quad \text{for} \quad \sin[2(\theta - \phi_0)] > 0;$$
$$m(\theta) = 0 \quad \text{elsewhere.}$$

This mask cancels the two quadrants in which the $\sin(2\theta)$ modulation is negative. The fact that only the difference $(\theta - \phi_0)$ intervenes in this expression shows that the mask must be integral with the linear polarizer (in our setup with the LCLV). By rotating the linear polarizer and the mask together by 180° or 360° and averaging the snapshots, we obtain (apart from a 0.5 factor that is due to the two black quadrants)

$$H_S(x, y) = (1/\pi)\left[\int_0^{2\pi} m(\theta)\sin(2\theta)\mathrm{d}\theta\right] \times \sin[\pi f_r(x^2 + y^2)],$$
$$H_S(x, y) = (2/\pi)\sin[\pi f_r(x^2 + y^2)]. \quad (7)$$

Note that without the mask this integral would be zero, i.e., we would obtain only the averaged bias. By numerical linear combination with H_c, with the proper $(i\pi/2)$ factor, we obtain the ideal PSF H_e.

The experimental setup is shown in Fig. 4, and the different PSF's obtained as functions of the transmittance of the LCLV are summarized in Table 1.

A collimated He–Ne 5-mW laser beam is focused through a microscope objective to a point that serves as a point source. The mask and the valve are mounted together on a rotation stage controlled by the microcomputer. The (slow and fast) axes of the LCLV (Meadowlark Optics LVR 0.7) are put at 45° to the edges of the mask; thus the axis of the linear polarizer resulting from the circular polarizer and the valve is aligned with the edges of the mask, in accordance with the definition of the mask's trans-

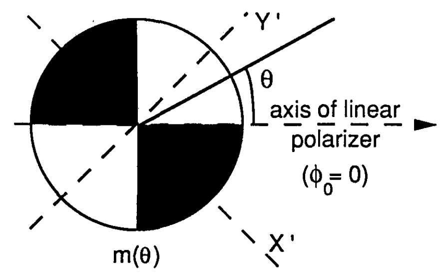

Fig. 3. Mask is put in the front focal plane of the system. X' and Y' are the axes of the LCLV. The circular polarizer and the LCLV (when set to a one- or three-quarter wave) give a linear polarizer whose axis ϕ_0 is at ±45° with X'.

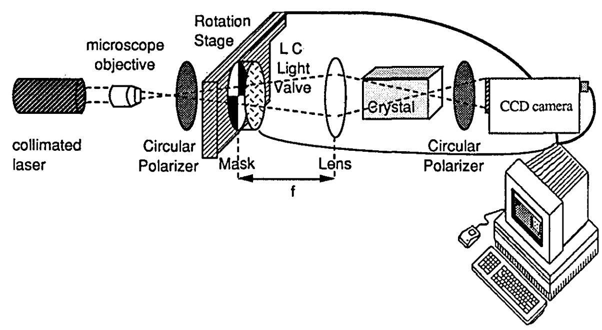

Fig. 4. Experimental setup showing the acquisition of the impulse response of the system.

Table 1. Different PSF Values, after Integration of the Snapshots, as Functions of the LCLV Delay[a]

LCLV	PSF
Standard scheme	
0	$H_c^+ = (1/2)[1 + \cos(\Psi)]$
$\lambda/2$	$H_c^- = (1/2)[1 - \cos(\Psi)]$
Numerical difference	$H_c = H_c^+ - H_c^- = \cos(\Psi)$
Our new scheme	
$\lambda/4$	$H_S^+ = (1/2)[1 + (2/\pi)\sin(\Psi)]$
$-\lambda/4$	$H_S^- = (1/2)[1 - (2/\pi)\sin(\Psi)]$
Numerical difference	$H_S = H_S^+ - H_S^- = (2/\pi)\sin(\Psi)$
Linear combination	$H_e = H_c + i(\pi/2)H_S = \exp(i\Psi)$

[a] H_e is the final, ideal PSF (with neither bias nor conjugate image), obtained by linear combination of the four experimental PSF's. Ψ is a shorthand notation for the phase $\pi f_r(x^2 + y^2)$.

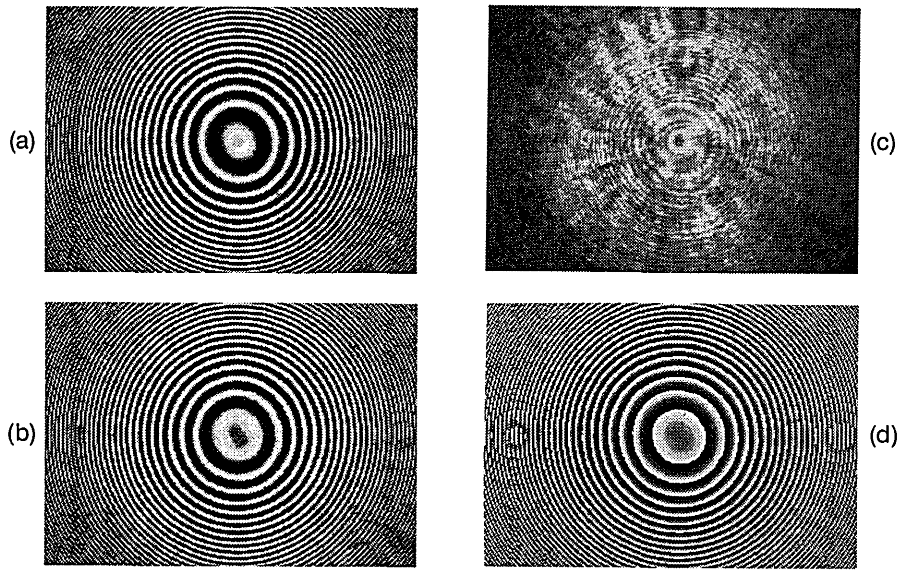

Fig. 5. (a), (b) Experimental results: integrated snapshots with the different PSF's giving (a) the real part H_c and (b) the imaginary part H_S of the ideal PSF H_e. Numerically calculated (c) modulus and (d) phase of the experimental PSF.

mittance. A standard 50-mm/1.8 Nikkor lens with the mask in its front focal plane images the point source into the rest of the system (a calcite 10 mm × 10 mm × 25 mm crystal, an output circular polarizer, and a CCD camera). We used calcite because of its high birefringence (0.17): the lateral resolution of the image reconstructed from a conoscopic hologram is proportional to the relative birefringence and to the length of the crystal.[2]

The images in Fig. 5 show the integrated snapshots with the cosine [Fig. 5(a)] and sine [Fig. 5(b)] PSF and the modulus [Fig. 5(c)] and phase [Fig. 5(d)]. Because of diffraction the sharp edges of the mask are smoothed, so that we lose a few central pixels (approximately the first fringe, i.e., half the first Fresnel zone). This effect can be reduced by taking diffraction into account in the calculation of the mask.

The image of the modulus is not uniform, which is mainly due to the nonsphericity of the point-source wave and to impurities in the system. Nevertheless, the phase image is of good quality, and the fit to a parabola of the unwrapped phase from the 10 central lines gave a standard deviation of 1.5% of a Fresnel zone (i.e., of 2π).

In conclusion, we have described a new method for obtaining both the real and the imaginary parts of the ideal PSF and presented some experimental results. This method will be used to restore the shapes of three-dimensional objects from their conoscopic holograms.

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