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A novel reconstruction method for weak-phase optical interferometry

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Abstract: Current optical interferometers are affected by unknown turbulent phases. We account for this lack of phase information by introducing aberration parameters, and solve the image reconstruction problem by minimizing an original regularized criterion.

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Interferometry allows one to reach the angular resolution that a hundred meter telescope would provide, using several ten meter telescopes. Because turbulence corrupts the object phases, one is led to form closure phases.We introduce phase calibration parameters which account for the missing phase information, and propose WISARD, an algorithm which alternately refines objet and phase parameters, in the spirit of self-calibration algorithms proposed by radio-astronomers. Our work is dedicated to process 3-telescope-interferometer data, although the method can be transposed to more than 3 telescope interferometer data.

1 APPROXIMATED DATA MODEL

We consider throughout this paper an interferometric array of 3 telescopes, although the method can be transposed to more than 3 telescope interferometer data. In this section, we design a metric which expresses the likelihood of the observables as a function of the object X and the noise statistics.

Direct Model In interferometry, the observables are time-averaged closures $\mathscr{C}_{mes}$ and visibility amplitudes A_{mes}. For n_a instants of measurement, there are n_a closures and $3\times n_a$ amplitudes. We assume here that only standard deviation on closures and amplitudes is supplied, and that all noises are uncorrelated, zero mean and Gaussian. The model consequently reads

$$\begin{cases} \mathscr{C}_{mes} = \Omega(HX) + \text{ Gaussian noise} \\ A_{mes} = |HX|, + \text{Gaussian noise} \end{cases} \tag{1}$$

with H the Fourier operator, and Ω the closure operator.

Myopic model The data likelihood yielded by this model is not convex [2] and has local minima, which makes minimization difficult [1]. Yet, there is another way to state the data model : it is possible to account for missing phase information through "myopic" aberration parameters. As one closure is measured per instant, instead of three visibility phases (one for each baseline), we explicitely account for this missing information by introducing 2 aberration parameters in the system.
Let us consider one triple including telescopes $1,2,3$. We suppose we have virtually measured all the visibility phases φ and introduce unknown aberration parameters β, which generate the null space of the closure operator Ω. If we define $\varphi = \arg V$, it follows :

$$\begin{cases} \varphi_{mes}(12) = \arg HX(12) + \beta_1 + \text{ Gaussian noise} \\ \varphi_{mes}(23) = \arg HX(23) + \beta_2 + \text{ Gaussian noise} \\ \varphi_{mes}(31) = \arg HX(31) - \beta_1 - \beta_2 + \text{ Gaussian noise} \end{cases} \tag{2}$$

With $\varphi_{mes} = \mathscr{C}_{mes}/3$ for each line, the sum of the three lines of system (2) is equal to $\mathscr{C}_{mes} = \Omega(HX) +$ Gaussian noise, which is exactly the first equation of system (3). The model then reads:

$$\begin{cases} \varphi_{mes} = \arg HX + \beta + \text{ Gaussian noise} \\ A_{mes} = |HX|, + \text{Gaussian noise} \end{cases} \tag{3}$$

We can then form virtually measured complex visibilities $V_{mes} = A_{mes} e^{\mathbf{i}\varphi_{mes}}$. From equation (3), we gather

$$V_{mes} = |HX| \exp \mathbf{i} [\arg HX + \beta] + N \quad (4)$$
$$= \mathscr{V}(X, \beta) + N \quad (5)$$

with N a complex noise. $\mathscr{V}(X, \beta)$ is the noiseless β-corrupted complex data model. The myopic data model (4) is equivalent to the direct data model (3).

Noise approximations We have shown [2] that model (4) yields a non-convex data-likelihood metric respect to X (i.e. with known aberration parameters β). Although it is possible to use circular approximations of the noise distribution N which are convex, they are not well adapted for optical interferometry. We hence use the elliptic convex approximation described in [2], which better fits the noise statistics and leads to a weighted quadratic criterion:

$$J_{ell}(X, \beta) = \|V_{mes} - \mathscr{V}(X, \beta)\|^2_{C_b}, \quad (6)$$

Properties of $J_{ell}(X, \beta)$ It can be easily shown that the criterion $J_{ell}(X, \beta)$ is the sum of n_a terms, each one involving only the measurements obtained at instant i. Hence, each J^i_{ell} depends only of 2 aberration parameters β^i_1 and β^i_2:

$$J_{ell}(X, \beta) = \sum_{i=1\ldots n_a} J^i_{ell}(X, \beta^i_1, \beta^i_2) \quad (7)$$

This is a consequence of the fact that, by definition, if the time between two instants is much greater than the turbulence coherent time (around 10ms), aberrations at two different instants are statistically independent.
Hence, $J_{ell}(X, \beta)$ is both **convex** with respect to X, **separable** with respect to β and designed from an approximated noise model which **accurately fits** the true one.

2 WISARD, AN ALTERNATING RECONSTRUCTOR

In this section, we describe our interferometric data reduction algorithm, WISARD, standing for Weak-phase (i.e. 3 or 4 pupil array case) Interferometric Sample Alternating Reconstruction Device.

Minimization strategy Although the criterion we have designed is convex for given aberrations, it is not convex for the whole unknown set, i.e. for both aberrations and object. In order to obtain a "good" solution with gradient-based minimization algorithms, it is useful, as empirically witnessed, on the one hand to estimate a reasonable initial guess from the visibility amplitude data; and on the other hand to incorporate the data gradually, starting with low frequencies end ending with high frequencies.

Global pattern In order to make use of the remarkable properties of $J_{ell}(X, \beta)$, we minimize it following an alternating pattern, i.e. we optimize it given the current aberrations with respect to the object, and then optimize with respect to the aberrations for the current object. The object step is a convex functional minimization under positivity constraint. It is performed by a BFGS-method (Broyden-Fletcher-Goldfarb-Shanno) software OP-VMLMB, designed by Eric Thiébaut[a] .
Since $J_{ell}(X, \beta)$ is separable for a given X, instead of optimizing for $2 \times n_a$ aberration parameters, we perform n_a separate optimizations of 2 parameters, each of which can be done simply by an exhaustive search on a fine grid.

Object Regularization Due to the poor spectral coverage, the object reconstruction, even with known aberrations, is an ill-posed inverse problem and must be regularized (see Refs. [3] and [4] for reviews on regularization). Whereas quadratic regularization tends to soften the edges of the reconstructed map, a quadratic-linear regularization (or L1-L2) is a good trade-off between obtaining a clean image and retrieving peaks [5, 6]. Among the several versions of a L1-L2 functional, we use the isotropic criterion developed by one of the authors [7].

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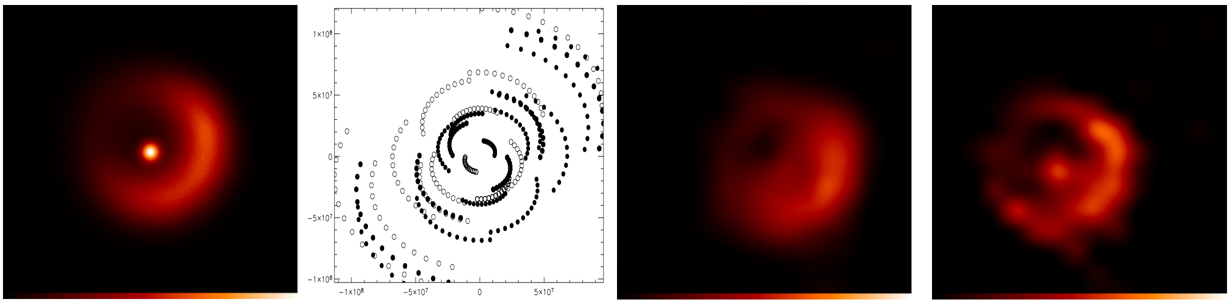
3 An Interferometric Imaging Beauty Contest

We took part in an international blind reconstruction contest, which aimed at comparing the performances of five different algorithms designed for synthesis imaging [8]. At the time, we were able to obtain only the L2 regularized map (Fig. 1.c) that was submitted to this contest. We present here the new results obtained by using L1-L2 regularization. The data sets have been produced by Christian Hummel, using the data reduction software OYSTER[b] and simulating a six-station Navy Prototype Optical Interferometer (NPOI). The image of the star with asymmetric shell shown in Fig. 1 was provided to Christian Hummel by Peter Tuthill. See [8] for more details on the contest.

Results As shown in Fig. 1 , we have retrieved satisfactorily the global structures of both objects. The central dot is not reconstructed with the quadratic regularization, but is retrieved with the L1-L2 one, although it is softer and slightly wider than the original.



(a) True object (b) Frequency coverage (c) Reconstructed (L2) (d) Reconstructed (L1-L2)

Fig. 1. Original object and reconstruction maps.

4 Conclusion

Although the reconstructed maps are satisfactory, we hope to get even closer by adapting WISARD to more-than-3-telescope arrays. Another crucial aspect is the extensive study of the criterion shape, mainly its behavior as a function of the uv-coverage. Finally, we hope we can further improve the reconstructions by using other regularization methods.

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[b]see http://www.sc.eso.org/˜chummel/oyster/oyster.html