# Reconstruction method for weak-phase optical interferometry

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Current optical interferometers are affected by unknown turbulent phases on each telescope. We account for this lack of phase information by introducing system aberration parameters, and we solve the image reconstruction problem by minimizing an original joint criterion in the aberrations and in the object. We validate this method by means of simulations. Tests on experimental data are under way. © 2005 Optical Society of America

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Optical interferometry allows one to reach the angular resolution that a 100 m telescope would provide using several 10 m telescopes. The interferograms of current instruments are affected by turbulence, which corrupts the recorded object phases, so one is led to form quantities that are turbulence independent such as phase closures. To cope with the missing phase information, we introduce phase calibration parameters to be estimated jointly with the observed object, and we propose a weak-phase interferometric sample alternating reconstruction device (WISARD) to perform this estimation. This algorithm combines, within a Bayesian framework, an alternating estimation of the object and phase parameters (in the spirit of self-calibration algorithms proposed by radioastronomers,[1] a recently developed noise model suited to optical interferometry data,[2] and an edge-preserving regularization[3] to deal with the sparsity of the data typical of optical interferometry. Our work is dedicated to processing data from three-telescope interferometers, yet the method can be easily extended to data from interferometers of more than three telescopes. We consider throughout this Letter an interferometric array of three telescopes $T_1$, $T_2$, and $T_3$, which are pointed at the same monochromatic source at $n_a$ different moments. The three frequencies measured at one moment $i$ are given by

$$\nu^i(12)=\frac{\overrightarrow{T_1T_2}}{\lambda},\qquad \nu^i(23)=\frac{\overrightarrow{T_2T_3}}{\lambda},\qquad \nu^i(31)=\frac{\overrightarrow{T_3T_1}}{\lambda}.$$

The basic observable of an interferometer without turbulence is the complex visibility, which can be measured from the fringe pattern obtained for each couple of telescopes belonging to the same array. We will call $V_0^i(12)$ the visibility corresponding to frequency $\nu^i(12)$ and arrange all these visibilities in a vector $\mathbf{V}_0$. With $X$ the unknown object and $H$ the Fourier operator, we have

$$\mathbf{V}_0(X)=HX. \tag{1}$$

We also define the visibility amplitudes and phases as $A=|HX|$ and $\varphi_0=\arg HX$, respectively.

When there is atmospheric turbulence, the instantaneous complex visibilities are affected by random optical path differences $\alpha^i(j)$ at each telescope $T_j$. One cannot measure $\mathbf{V}_0$, only its turbulence-corrupted value $\mathbf{V}$:

$$V^i(12)=V_0^i(12)\exp i[\alpha^i(2)-\alpha^i(1)],$$

$$V^i(23)=V_0^i(23)\exp\{i[\alpha^i(3)-\alpha^i(2)]\},$$

$$V^i(31)=V_0^i(31)\exp\{i[\alpha^i(1)-\alpha^i(3)]\}. \tag{2}$$

If we define $\varphi=\arg\mathbf{V}$, it follows that

$$\varphi^i(12)=\varphi_0^i(12)+\alpha^i(2)-\alpha^i(1),$$

$$\varphi^i(23)=\varphi_0^i(23)+\alpha^i(3)-\alpha^i(2),$$

$$\varphi^i(31)=\varphi_0^i(31)+\alpha^i(1)-\alpha^i(3). \tag{3}$$

Actually, even the corrupted visibility phases $\varphi=\arg\mathbf{V}$ are difficult to measure, because they fluctuate in a fast and random way. However, the visibility amplitudes are not affected and can still be measured: $A=|\mathbf{V}_0|=|\mathbf{V}|$. Moreover, we find that the turbulent phasers cancel out[4] in

$$\mathcal{C}^i=\arg V^i(12)+\arg V^i(23)+\arg V^i(31),$$

$$=\varphi^i(12)+\varphi^i(23)+\varphi^i(31),$$

$$=\varphi_0^i(12)+\varphi_0^i(23)+\varphi_0^i(31),$$

or in a vector formulation $\boldsymbol{\mathcal{C}}=\Omega\mathbf{V}_0=\Omega\mathbf{V}$. $\Omega$ is the so-called closure operator.

 

We will now design a metric that expresses the likelihood of the observables as a function of $X$ and the noise statistics. The observable are time-averaged closures $\mathcal{C}_{mes}$ and visibility amplitudes $A_{mes}$ (actually, most interferometers provide squared visibilities from which we extract amplitudes). We assume here that only standard deviation on closures and amplitudes will be supplied and that all the noises are uncorrelated, zero mean, and Gaussian. The model consequently reads

$$\mathcal{C}_{mes} = \boldsymbol{\Omega}(HX) + \text{Gaussian noise},$$

$$A_{mes} = |HX| + \text{Gaussian noise}. \quad (4)$$

The data likelihood yielded by this model is not convex[5] and has local minima, which makes minimization difficult.[6] Yet, there is another way to state the data model: It is possible to account for missing phase information through myopic aberration parameters. Because of random delays $\alpha$ we measure one closure per instant, instead of three phases. In other words, two pieces of information per triple are missing. We can account for this missing information by introducing two aberration parameters in the system.

Let us consider system (3). We suppose we have measured virtually all the corrupted phases $\varphi$ and introduce the turbulent optical path delays through unknown aberration parameters $\beta$, which generate the null space of the closure operator $\Omega$. If we define $\varphi = \arg \mathbf{V}$, it follows that

$$\varphi^i_{mes}(12) = \varphi^i_0(12) + \beta^i_1 + \text{Gaussian noise},$$

$$\varphi^i_{mes}(23) = \varphi^i_0(23) + \beta^i_2 + \text{Gaussian noise},$$

$$\varphi^i_{mes}(31) = \varphi^i_0(31) - \beta^i_1 - \beta^i_2 + \text{Gaussian noise}, \quad (5)$$

$$\varphi_{mes} = \arg HX + \beta + \text{Gaussian noise}. \quad (6)$$

If we set $\varphi^i_{mes}(12) = \varphi^i_{mes}(23) = \varphi^i_0(23) = \mathcal{C}^i_{mes}/3$, the sum of the three lines of system (5) is equal to

$$\mathcal{C}^i_{mes} = \varphi^i_0(12) + \varphi^i_0(23) + \varphi^i_0(31) + \text{Gaussian noise},$$

which is exactly the first equation of system (4). With Eq. (6) and the second equation of system (4), we define

$$\mathbf{V}_{mes} = A_{mes} e^{i\varphi_{mes}},$$

$$\mathcal{V}(X,\beta) = |HX| \exp[i(\arg HX + \beta)], \quad (7)$$

and gather $\mathbf{V}_{mes} = \mathcal{V}(X,\beta) + N$, with $N$ being a complex noise. $\mathcal{V}(X,\beta)$ is the noiseless $\beta$-corrupted complex data model. Myopic data model (7) is equivalent to direct data model (4).

We have shown[5] that model (7) yields a nonconvex data-likelihood criterion. Although it is possible to use circular approximations of the noise distribution $N$ (Ref. 7) that are convex, they are not well adapted for optical interferometry. We hence use the elliptic convex approximation described in Ref. 5, which better fits the noise statistics and can be stated as

$$J_{ell}(X,\beta) = \|\mathbf{V}_{mes} - \boldsymbol{\mathcal{V}}(X,\beta)\|^2_{C_b}. \quad (8)$$

It can be easily shown that the criterion $J_{ell}(X,\beta)$ is the sum of $n_a$ terms, each involving only the measurements obtained at instant $i$. Hence, each $J^i_{ell}$ depends only on two aberration parameters $\beta^i_1$ and $\beta^i_2$:

$$J_{ell}(X,\beta) = \sum_{i=1\ldots n_a} J^i_{ell}(X,\beta^i_1,\beta^i_2). \quad (9)$$

This is a consequence of the fact that, by definition, if the time between two instants is much greater than the turbulence coherent time (around 10 ms), aberrations at two different instants are statistically independent. Hence, $J_{ell}(X,\beta)$ is convex with respect to $X$, separable with respect to $\beta$, and designed from an approximated noise model that accurately fits the true one.

Here, we describe our interferometric data reduction algorithm WISARD (for three or four pupil array cases). Although the criterion we designed is convex for given aberrations, it is not convex for the whole unknown set, i.e., for both aberrations and objects. To obtain a good solution with gradient-based minimization algorithms, it is useful, as empirically witnessed, on the one hand to estimate a reasonable initial guess from the visibility data and on the other hand to incorporate the data gradually, starting with low frequencies and ending with high frequencies. To make use of the remarkable properties of $J_{ell}(X,\beta)$, we minimize it following an alternating pattern; i.e., we optimize it given aberrations with respect to the object and then optimize the aberrations for the current object.

The object step is a convex functional minimization under a positivity constraint. It is performed by Broyden–Fletcher–Goldfarb–Shanno method software, OP-VMLMB designed by Eric Thiébaut.[8] Since $J_{ell}(X,\beta)$ is separable for a given $X$, instead of optimizing for $2n_a$ aberration parameters, we perform $n_a$ separate optimization of two parameters that can be done simply by an exhaustive search on a fine grid. The pattern of WISARD is described in Fig. 1.

Due to the poor spectral coverage, the object reconstruction, even with known aberrations, is an ill-posed inverse problem and must be regularized (see Refs. 9 and 10 for reviews on regularization). Whereas quadratic regularization tends to soften the

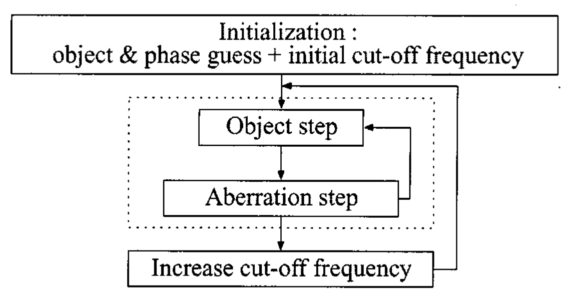


Fig. 1. WISARD algorithm loop.

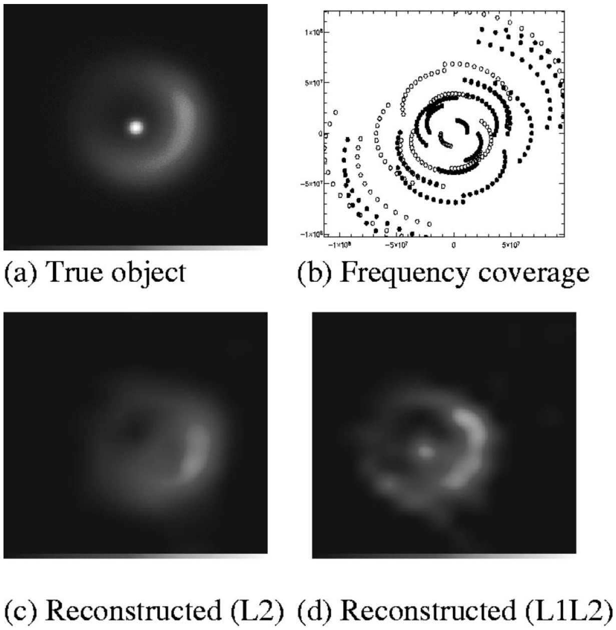


Fig. 2. Original object and reconstruction maps.

edges of the reconstructed map, a quadratic-linear regularization (or L1–L2) is a good trade-off between obtaining a clean image and retrieving peaks.[11,12] Among the several versions of an L1–L2 functional, we use the isotropic criterion developed by one of the authors.[3] We took part in an international blind reconstruction contest, which aimed at comparing the performance of five different algorithms designed for synthesis imaging. Only the L2-regularized map [Fig. 2(c)] was submitted to this contest,[13] and our method has evolved toward a less-case-dependent procedure since then. We present here the new results obtained by using L1–L2 regularization.

The data sets were produced by Christian Hummel using the data reduction software OYSTER[14] and simulating a six-station Navy prototype optical interferometer (NPOI). The image of the star with an asymmetric shell shown in Fig. 2 was provided to Christian Hummel by Peter Tuthill. As shown in Fig. 2, we retrieved satisfactorily the global structures of both objects. The central dot is not reconstructed with the quadratic regularization but is retrieved with L1–L2 regularization, although it is softer and slightly wider than the original.

Although the reconstructed maps are satisfactory, we hope to get even closer by adapting WISARD to arrays of more than three telescopes. Another crucial aspect is the extensive study of the criterion shape, mainly its behavior as a function of UV coverage. Finally, we hope we can further improve the reconstructions by using other regularization methods.

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