Variations on a Hartmann theme

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Abstract. We explore modifications of the classical Hartmann wavefront sensing technique that can be used to improve its accuracy, dynamic range, and spatial resolution. We describe a differential sensor with variable sensitivity. We review the use of various possible Hartmann masks and discuss their interferometric properties. We propose the use of Fourier analysis and show its relationship to moiré methods. We finally envisage the possibility of mapping both the slope and the total curvature (Laplacian) of the wavefront with the same setup.

Subject terms: adaptive optical components; wavefront sensors; Hartmann test; optical figure testing; interferometry; Fourier analysis; moiré deflectometry; curvature sensing.

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1. INTRODUCTION
Optical testing is usually done with interferometers, using a laser source and a reference wave. However, in some cases it becomes problematic or cannot be done at all. An example is the testing of very large optics such as telescope mirrors. The distance to travel becomes so large that the effects of vibrations and air turbulence cannot be overcome. Another example is sensing an atmospherically distorted wavefront, when no undistorted reference wave is available or when the source is incoherent (broadband and possibly extended).

In these cases, the current approach consists of measuring local wavefront slopes and reconstructing the wavefront from its slopes. This can be done either with a shearing interferometer or with a Hartmann sensor. The latter is usually preferred because it is achromatic. Achromatic shearing interferometers have also been used. As we shall see, a Hartmann sensor can also be considered as an achromatic shearing interferometer.

Although widely used for its merits, the technique has not evolved much since Hartmann. To date, the only major improvement made to the original Hartmann test is the use of lenslets, proposed by Shack. Here, we describe other modifications of the original test, which depending upon the specific application could be used to improve the performance of the method.

2. DIFFERENTIAL HARTMANN SENSOR
Let us go back to the original Hartmann technique in which a mask with holes is used in the telescope pupil. Each hole isolates a ray of light. The test consists of recording the ray impacts in a plane slightly before the focal plane. If optics were perfect, the recorded spots would be exactly distributed as the holes in the mask, but on a smaller scale. Owing to aberrations, light rays are deviated from their ideal direction, producing spot displacements. The amount of displacement is a measure of the deviation of the ray, which is also the deviation of the local wavefront slope since rays are normal to the wavefront.

It is easy to see that if the ray impacts are recorded on the other side of the focal plane, the displacements occur in the opposite direction. Hence, by comparing spot displacements on each side of the focal plane one can double the test sensitivity. Complete symmetry can be achieved by putting a converging lens L in the reference focal plane that would reimage the telescope pupil symmetrically behind the focal plane. By “reference focal plane” we mean the focal plane of the spherical wave used as the reference with respect to which the aberrations are measured. This is shown in Fig. 1. Identical but opposite deviations Δx are observed in plane P1 and P2 at a distance l from the reference focal plane. For telescopes with a long focal length f, the convergence of L is so small that it produces only a minor correction.

By changing the convergence of lens L, one can easily reimagine planes P1 or P2 on a detector beyond the focal plane at a distance determined by its size. Because L is in the telescope focal plane, its aberrations do not add but only affect the sensor response linearity. A very convenient scheme consists of using a variable convergence device (VCD), such as a variable curvature mirror (Fig. 2). In its rest position the mirror is flat, and optics must be used after the mirror to reimagine the telescope pupil onto the detector. The mirror can then be activated to defocus the pupil image in either direction, thus producing symmetric spot displacements. By rapidly switching defocus one can observe the spots oscillate along the tilt direction, thus materializing the wavefront slope gradient field. Since the aberrations of the reimaging optics do not contribute to the modulation, they again only affect the sensor response linearity. A nice characteristic of this scheme is that its sensitivity can be easily modified by varying the amount of defocus introduced; hence, the signal can always be adjusted to match the detector dynamic range. This can be very important in applications such as adaptive optics because atmospheric wavefront degradation is highly variable.

In the case of telescope mirror figure testing, very accurate measurements can be obtained by mounting the VCD on a high
Fig. 1. The wavefront slope at points $x_0$ produces opposite but otherwise identical ray displacements $\Delta x$ in planes $P_1$ and $P_2$.

Fig. 2. A variable curvature mirror can be used to defocus in either direction the telescope pupil reimaged onto the detector.

precision $x,y,z$ translation stage. Indeed, a VCD modulator behaves as a position sensitive device. When a light ray hits the device, it is periodically deviated with an amplitude proportional to the distance between the impact point and the VCD optical center. For a given $z$ position of the VCD along the optical axis, one can find its $x,y$ position that cancels out the spot oscillation. This gives the impact point coordinates. By repeating this measurement for all the spots, one gets a spot diagram of the ray impacts in plane $z$ from which the local wavefront slopes can be determined. This nulling technique is of course insensitive to any nonlinearity that might be introduced by the VCD or the following reimaging optics. Moreover, one can repeat the measurements at different positions $z$ along the optical axis and trace the rays in the neighborhood of the focusing point. This is especially useful when testing a parabolic or hyperbolic mirror at the center of curvature. In this case, one can use a Hartmann mask with holes on concentric circles and find the $z$ position that minimizes the spot oscillations for each circle, thus determining the converging point for each concentric zone. This will determine with great accuracy the amount of spherical aberration observed and hence the conical constant of the mirror. The dynamic range of the method is only limited by the range of the $x,y,z$ translation stage. It is so large that no null lens need be used.

3. VARIETY OF POSSIBLE HARTMANN MASKS

We have seen that Hartmann sensing actually consists of comparing the illumination in the masked pupil with the illumination in the defocused image of the pupil or the illumination in two symmetrically defocused pupil images. For example, we are comparing the illumination in the pupil plane with the illumination at some distance before or after the pupil plane. If this distance $\Delta z$ is small, then the variation $\Delta I$ of illumination $I$ is given by the irradiance transport equation

$$\Delta I = -(\nabla I \cdot \nabla W + I \nabla^2 W)\Delta z,$$

where $W(x,y)$ is the wavefront surface and $\nabla$ is the $\partial/\partial x, \partial/\partial y$ operator. Equation (1) contains two terms. The first term $\nabla I \cdot \nabla W$ represents the variation of illumination caused by a transverse shift of the nonuniform illumination ($\nabla I \neq 0$) in the pupil plane. This shift is proportional to the local wavefront slope $\nabla W$, which deviates the direction of propagation. This term may be called a prism term. The second term $I \nabla^2 W$ represents the variation of illumination caused by convergence or divergence of the beam and is proportional to the local total wavefront curvature or Laplacian $\nabla^2$. This term may be called a lens term. Ichikawa et al. have recently experimentally demonstrated wavefront phase retrieval from irradiance measurements using this transport equation. We have recently shown its relevance to incoherent wavefront sensing techniques.

Equation (1) basically shows that one can measure both the local wavefront slopes and the local wavefront Laplacians. By putting a mask with holes in the pupil plane, one produces strong variations of the illumination (strong gradients $\nabla I$), thus maximizing the sensitivity to wavefront slopes. We have a Hartmann sensor. On the other hand, one can just leave the pupil without a mask and still measure the local wavefront Laplacian. Since the telescope pupil itself acts as a mask, we are still sensitive to wavefront slopes at the pupil edge. Hence, one can reconstruct the wavefront from its Laplacian by solving the Poisson equation with edge slopes as Neumann boundary conditions. We have a curvature sensor. Clearly, depending upon the application, one can envisage a large variety of pupil masks. We review some of them here.

Our first example will be the Ronchi ruling. By putting a periodic bar pattern in the pupil plane, one can observe how the pattern deforms along the propagating beam. This is indeed the essence of the so-called Ronchi test. It is sensitive to wavefront slopes in a direction perpendicular to the bars. After propagation over a distance $\Delta z$, bars along the $y$ direction are locally distorted and shifted in the $x$ direction by an amount

$$\Delta x = \left( \frac{\partial W}{\partial x} \right) \Delta z.$$

To observe wavefront slopes in another direction, one has to rotate the ruling in this plane. A pair of crossed Ronchi rulings is a Hartmann mask. It indeed gives the wavefront slope along two directions.

A Ronchi ruling also can be considered as a diffraction grating. A grating with period $a$ diffracts light into waves propagating at angles that are multiples of $\lambda/a$. The periodic pattern observed along the beam is the result of interference between these waves. The first harmonic is a fringe pattern produced by waves interfering at an angle $\lambda/a$. At a distance $\Delta z$, the interfering wavefronts are laterally sheared by an amount

$$\Delta x = \left( \frac{\lambda}{a} \right) \Delta z.$$
Hence, the fringe phase is shifted by an amount \( \delta \phi = (\partial \phi/\partial x) \Delta x \), or in terms of the wavefront surface \( W = (\lambda/2\pi)\phi \),

\[
\delta \phi = \left( \frac{2\pi}{\lambda} \right) \left( \frac{\partial W}{\partial x} \right) \Delta x .
\]

Putting Eq. (3) into Eq. (4) gives

\[
\delta \phi = \left( \frac{2\pi}{a} \right) \left( \frac{\partial W}{\partial x} \right) \Delta z ,
\]

which means that the fringes are laterally shifted by an amount

\[
\delta x = \left( \frac{a}{2\pi} \right) \delta \phi = \left( \frac{\partial W}{\partial x} \right) \Delta z ,
\]

which is a result identical to Eq. (2). Hence, a Hartmann test with a periodic mask such as a Ronchi ruling can be considered to be an achromatic lateral shear interferometer. The distorted bar pattern produced by the test and the distorted interference fringes produced by a lateral shear interferometer are essentially the same. Another interesting mask is the Fresnel mask with alternatively opaque and transparent concentric Fresnel zones. Such a mask is appropriate to measure wavefront slopes in a radial direction. One can similarly show that after propagation the mask pattern deforms, producing the same pattern an achromatic radial shear interferometer would produce.

The masks considered so far transmit only part of the light. At low light level, this can be detrimental. In this case one should consider using a phase screen as a Hartmann mask. As light propagates away, the spatial phase modulation introduced by the mask is gradually converted into an amplitude modulation that can be detected. For instance, one can use a sinusoidal phase grating with period \( a \). In a collimated beam, the phase modulation of a sinusoidal grating is entirely converted into a pure amplitude modulation over a distance \( \frac{a^2}{2} / (\alpha^2/\lambda) \), where 100% contrast fringes are observed. While still operating in the near-field regime for the larger scale wavefront errors we want to measure, this is no longer near-field diffraction for the mask. Hence, by again comparing the amplitudes in two opposite distortions as described in Sec. 2 and subtracting their average distance. By recording two arrays of spots with opposite distortions as described in Sec. 2 and subtracting the observed phase distortion, one can also double the sensitivity but also eliminate phase errors produced by the mask irregularities.

One could measure wavefront radial slopes by putting Fresnel phase screens in the pupil plane or even use a simple diffuser (ground glass). After propagation a speckle pattern is observed. By comparing the speckle patterns in two oppositely defocused pupil images one can map the displacement field and hence the wavefront gradient field. The technique of measuring speckle displacement fields is currently used to measure deformations of materials and is called electronic speckle pattern interferometry (ESPI).

**4. FOURIER TRANSFORM METHODS**

We have seen that the distorted pattern produced by a periodic Hartmann mask can be considered as a distorted fringe pattern. As a consequence Fourier transform techniques that have been successfully applied to interferogram analysis \(^9\)\(^-\)\(^11\) also apply to Hartmann data analysis. For instance, let us consider a standard Hartmann mask made of a two-dimensional periodic array of holes. After propagation, the detector records a distorted array of spots. The Fourier transform of this irradiance distribution is a two-dimensional periodic array of harmonics each being widened by the distortion. The technique consists of selecting one of the harmonics through a filtering window and taking the inverse Fourier transform. The phase of this transform maps the local phase distortion of the periodic array and is therefore a measure of the wavefront slope in the direction of the harmonic (modulo \(2\pi\)). The technique will work with any two-dimensional periodic mask, such as crossed rulings or crossed phase gratings. Such an application would considerably increase the dynamic range and/or the spatial resolution of Hartmann sensors. Indeed, most current centroiding methods require the absolute displacement of Hartmann spots to be smaller than half the average spot interval. With Fourier transform techniques, this condition is no longer required. Phase unwrapping only requires the difference between two adjacent spot displacements to be smaller than half their average distance. By recording two arrays of spots with opposite distortions as described in Sec. 2 and subtracting the observed phase distortion, one can not only double the sensitivity but also eliminate phase errors produced by the mask irregularities.

Whereas the phase distortion of the periodic array is a measure of the local wavefront slope, the similarly obtained amplitude distortion is a measure of the local wavefront Laplacian (see Sec. 3). Hence, by again comparing the amplitudes in two oppositely defocused pupil mask images (Sec. 2), one can also map the wavefront Laplacian. Apart from the additional use of a pupil mask, this is essentially identical to the curvature sensing method.\(^6\)\(^,\)\(^7\) By measuring both the local wavefront slopes and the local wavefront Laplacian, one doubles the number of reconstructed points on the wavefront. Indeed, a Hartmann mask with period \( a \) is insensitive to a sinusoidal wavefront error of period \( 2a \) in phase with the mask, since the sinusoidal error is sampled only at extrema where the slope is zero. This is the cutoff frequency of the Hartmann mask. However, at these very same sample points, the wavefront curvature is an extremum. Hence, by measuring the wavefront Laplacian also, one becomes fully sensitive to wavefront errors at this spatial frequency. A widely used wavefront reconstruction method consists of computing the wavefront Laplacians from the wavefront slopes and solving the Poisson equation. Figure 3 displays the array of Laplacians computed from the slopes together with the array of

![Fig. 3. Possible wavefront sampling. Crosses: measured slopes. Closed circles: measured Laplacians. Open circles: Laplacians estimated from slopes.](http://opticalengineering.spiedigitallibrary.org/)
independently observed Laplacians, showing that the number of sampling points is doubled.

For adaptive optics applications where speed is an important requirement, one may wish to use an analog method to do the same thing. This can be done by putting a periodic mask with the same periodicity in the plane of the observed distorted pattern. One then observes a moiré pattern that reveals the wavefront distortion. This is equivalent to moiré deflectometry. To avoid any loss of light, this can be done after image intensification. The periodicity of a CCD or a reticon detector array can also be used to produce the same result provided it matches the mask periodicity. The pattern observed is the same as that produced by a lateral shear interferometer when the angle between the two interfering wavefronts is brought down to zero. Using the modulating technique discussed in Sec. 2, one can record a modulated signal with an amplitude proportional to the wavefront slope. The phase is 0° or 180°, according to the sign of the slope. When the slope is large the signal will saturate and start to wrap around. This can be avoided by automatically adjusting the amplitude of the modulation to keep the signal in the linear range. A second channel can be used to record the wavefront slopes in a perpendicular direction and a third channel without a mask can be used to record the wavefront Laplacian as well as the normalizing intensity.

In Sec. 3 it was mentioned that a Fresnel mask produces patterns identical to those of a radial shear interferometer. In this case one could similarly use a second Fresnel mask with alternately opaque and transparent Fresnel zones in the detector plane. By doing so, one produces the equivalent of a Fresnel transform. Indeed, it has been noted that Fourier transforms can be replaced with Fresnel transforms when dealing with radial shear interferograms. The observed moiré pattern is identical to the fringe pattern one would observe in a radial shear interferometer when the two centers of curvature of the interfering wavefront coincide. Again using the modulation technique described in Sec. 2, one can record a modulated signal proportional to the wavefront radial slope.

5. CONCLUSION

We hope we convinced the reader that Hartmann’s original idea is very fruitful and that all its possibilities have not yet been explored. More specifically: (1) We have described a differential Hartmann technique in which the spot displacement can be inverted. The difference between the two spot displacements is a measure of the wavefront slope independent of the mask irregularities. The sensitivity can be modulated at will. (2) We have shown that one can perform Hartmann testing with a wide variety of masks—not only holes or lenslets, but also crossed rulings or gratings, Fresnel masks, or even diffusers—and take advantage of their diffraction properties. (3) We have discussed the equivalence of Hartmann sensing and shearing interferometry and shown that Fourier transform techniques developed for interferogram reduction also apply to Hartmann images, vastly extending the spatial resolution and the dynamic range of the method. (4) Finally, we have shown that both the local wavefront slope and the local wavefront curvature (Laplacian) can be mapped with the same optical setup, doubling the number of reconstructed points on the wavefront.

6. ACKNOWLEDGMENTS

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7. REFERENCES


François Roddier received his doctorate degree from the University of Paris in 1964. He founded the Astrophysics Department of the University of Nice in 1966 and headed the department until 1981. He worked as a staff astronomer at the National Optical Astronomy Observatories in Tucson from 1984 to 1988. He is now with the Institute for Astronomy at the University of Hawaii. Dr. Roddier’s main interests have been, successively, atomic beam spectroscopy and the solar gravitational red shift; solar Doppler velocimetry with sodium resonance cells, which led to helioseismology; optical propagation through turbulence; interferometric image reconstruction in optical astronomy; and adaptive optics and wavefront sensing.

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